



# Independent Study | in Idaho

## Math 144 Analytic Trigonometry

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# Course Guide

Independent  
Study | in Idaho

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## **Math 144** **Analytic Trigonometry**

University of Idaho  
1 Semester-Hour Credits

**Prepared by:**

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## Math 144: Analytic Trigonometry

1 Semester-Hour Credits: UI

### Welcome!

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Whether you are a new or returning student, welcome to the Independent Study in Idaho (ISI) program. Below, you will find information pertinent to your course including the course description, course materials, course objectives, as well as information about assignments, exams, and grading. If you have any questions or concerns, please contact the ISI office for clarification before beginning your course.

### Policies and Procedures

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Refer to the ISI website at [www.uidaho.edu/isi](http://www.uidaho.edu/isi) and select *Students* for the most current policies and procedures, including information on setting up accounts, student confidentiality, exams, proctors, transcripts, course exchanges, refunds, academic integrity, library resources, and disability support and other services.

### Course Description

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Trigonometric functions, inverse functions, applications.

Prerequisite: Sufficient score on SAT, ACT, or math placement test. Students may qualify by enrolling concurrently in Math 143 or Math 170. Required test scores can be found at <http://www.uidaho.edu/registrar/registration/placement>.

UI students: Not open for credit to students who have previous high school or college credit in trigonometry. Polya Math Center unavailable for ISI students.

*Required: Internet access*

*Recommended: headphones*

*16 graded assignments, 3 proctored exams*

Students may submit up to 1 unit at a time/1 unit per week. A unit is a set of lessons preceding an exam. For example, lessons 1A through 6 comprise unit 1. Note that completing 1 chapter per week is a very demanding pace. It is more realistic to complete two lessons per week and prepare for each test over the course of a few days before taking it. Before taking exams, students **MUST** wait for grades and feedback on assignments, which may take up to three weeks after date of receipt by the instructor.

ALL assignments and exams must be submitted to receive a final grade for the course.

### Course Materials

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#### Required Course Materials

Trigsted, Kirk, *MyLab Math Series for Math 144*, 3<sup>rd</sup> ed. (MyLab Student Access Kit (code)), Pearson, 9781256698180. Access to and purchase information for MyLab will be provided in Canvas upon course registration.

### Course Delivery

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All ISI courses are delivered through Canvas, an online management system that hosts the course lessons and assignments and other items that are essential to the course. Upon registration, the student will receive a *Registration Confirmation Email* with information on how to access ISI courses online.

## Course Goals

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The primary purpose of Trigonometry is to improve your skills and competency in trigonometry to prepare you for calculus. Another goal is to help you develop your mathematical learning skills so that you will be more confident in future mathematical courses.

## Course Objectives

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After completing Math 144, the student should be able to do the following without the use of a calculator:

- Understand the right triangle definitions of the trigonometric functions
- Understand the unit circle definitions of the trigonometric functions
- Evaluate trigonometric functions of angles belonging to the  $\frac{\pi}{3}$ ,  $\frac{\pi}{4}$ , and  $\frac{\pi}{6}$  families
- Sketch the graph of functions of the form  $y = A\sin(Bx + C) + D$  and  $y = A\cos(Bx + C) + D$
- Understand the graph of the tangent function and its properties
- Understand the graph of the cosecant and secant functions and their properties
- Understand the graphs of the inverse sine function, inverse cosine function, and inverse tangent function
- Evaluate expressions involving inverse trigonometric functions
- Verify trigonometric identities
- Solve trigonometric equations

## Lessons

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### Overview

This course will cover three chapters (Ch 1, Ch 2, and Ch 3). There are a total of 16 homework assignments (one written and 15 online). Your homework score will be computed at the end of the course. All assignments will be averaged and the average will be scaled to 10% of your total grade. You can think of the average as being scaled to 10 points.

Each lesson may include the following components:

- lesson objectives
- reading assignments
- important terms
- lecture
- written assignment, project, or activity

### Study Hints:

- The best way to be successful in this course is to complete all of the assigned work. Carefully read the eText and watch the videos and animations **before** attempting the corresponding homework problems. Take notes in the space provided on the lesson pages and complete the homework when indicated.
- Carefully copy down and work out every homework problem. Keep a notebook of the homework problems.
- Keep a copy of every assignment submitted.

- Complete all reading assignments.
- Set a schedule allowing for course completion one month prior to your personal deadline. An *Assignment Submission Log* is provided for this purpose.
- Web pages and URL links in the World Wide Web are continuously changing. Contact your instructor if you find a broken Web page or URL.

Refer to the **Course Rules** in Canvas for further details on assignment requirements and submission.

## **Exams**

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- The exams are cumulative.
- Calculators are **not allowed** during exams.
- No notes of any kind are allowed during tests, although a sheet of formulas will be provided during Exam 3.
- **NO DEVICES** which are capable of transmitting or receiving data, including but not limited to watches, phones, tablets, iPods, and calculators, may be on your person during the exam. Any such items are expected to be left at home or left with the proctor. Failure to do so will result in a zero on the exam and possibly a failing grade for the course.
- You must wait for grades and comments on assignments prior to taking subsequent exams.
- For your instructor's exam guidelines, refer to the **Course Rules** in Canvas.
- Refer to *Grading* for specific information on assignment/exam points and percentages.

There will be one exam after the completion of each chapter (see the Notebook Table of Contents for a list of sections covered) for a total of three exams throughout the semester. Each exam is worth 100 points. Each exam will cover material from the entire course leading up to the exam. Therefore, exam 3 will cover the entire course. Students may take only one exam per day.

### **Proctor Selection/Scheduling Exams**

All exams require a proctor unless an exam is self-administered. To submit your *Proctor Information Form* online, visit the ISI website and select *Forms, Proctor Information Form*. Submit this form at least two weeks before your first exam. Refer to *Students, Assignments and Exams* on the ISI website for information on acceptable and unacceptable proctors.

## **Grading**

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The course grade will be based upon the following considerations:

### **Assignment Percentage**

Homework Total 10%

<b><u>Exam</u></b>	<b><u>Percentage</u></b>
Exam 1	30%
Exam 2	30%
Exam 3	30%
Total	90%

Your course grade will be based on the percentage that you have earned as follows:

90% guarantees an A

80% guarantees a B

70% guarantees a C

60% guarantees a D

The final course grade is issued after all assignments and exams have been graded.

## **Academic Honesty**

Acts of academic dishonesty, including cheating or plagiarism are considered a very serious transgression and may result in a grade of F for the course.

Students are expected to maintain Academic Honesty in all their work. Collaboration is encouraged on homework assignments. All tests are considered individual work and must be completed without unauthorized assistance of any kind, including the help of other students, tutors, notes, or calculators. All test materials and scratch paper are to be turned in with the test paper and attempting to bring test work out of the testing area and/or share that work with other students is considered cheating.

The University of Idaho has defined acceptable behavior in the Student Code of Conduct Article II.A-1 – Academic Dishonesty [rev. 7-98, 7-05, 7-14, ed. 7-09]. The following summarizes relevant points related to your math course:

- **Because academic honesty and integrity are core values at a university, the faculty finds that even one incident of academic dishonesty may merit expulsion.**
- **Cheating on classroom or outside assignments, examinations, or tests is a violation of this code.**
- Plagiarism, falsification of academic records, falsification of records and the acquisition or use of test materials without faculty authorization are considered forms of academic dishonesty and, as such, are violations of this code.
- Instructors and students are responsible for maintaining academic standards and integrity in their classes. Consequences for academic dishonesty may be imposed by the course instructor. Such consequences may include but cannot exceed a grade of "F" in the course.

The full text of the Student Code of Conduct may be found at

<http://www.uidaho.edu/DOS/judicialaffairs/studentcodeofconduct/Student%20Code%20of%20Conduct>

## **About the Course Developer**

Theresa Allen has degrees in chemistry, math, and chemical engineering. She has been teaching math at the University of Idaho since 1999. Her teaching experience includes intermediate algebra, precalculus, trigonometry, finite math, introductory Calculus, and math for liberal arts majors.

Dr. Allen hopes that every student in this course will learn something new about the subject and about him/herself as a learner. The subject matter appears in nature and man-made contexts in exciting ways. She looks forward to students finding examples of the topic in the wider world.

## **Contacting Your Instructor**

Instructor contact information is posted on your Canvas site under *Course Rules*.

## Math 144 Grade Summary

Enter your scores in the charts below, as percentages. Remember that you may only take one test in a day. See the Syllabus for the requirements.

### Homework (HW)

<i>Homework</i>	<i>Possible</i>	<i>Earned</i>
Introduction	100	
Section 1.1	100	
Section 1.3	100	
Section 1.4	100	
Section 1.5	100	
Section 1.6	100	
Section 2.1A	100	
Section 2.1B	100	
Section 2.2	100	
Section 2.3	100	
Section 2.4	100	
Section 2.5	100	
Section 3.1	100	
Section 3.2	100	
Section 3.3	100	
Section 3.5	100	
<b>Total</b>	<b>1600</b>	

Divide the HW total by 160 and write the result here: \_\_\_\_\_ Put this result in the table below: \_\_\_\_\_

### Grading Scale

- A: 90 – 100 % (pts)
- B: 80 – 89% (pts)
- C: 70 – 79% (pts)
- D: 60 – 69% (pts)
- F: Below 50% (pts)

<b>TOTALS</b>	<i>Possible</i>	<i>Earned</i>
HW (average of all 16)	10	
Test 1	30	
Test 2	30	
Test 3	30	
Sum of the scores above	100	



## Math 144 Grade Summary Example

This person earned an “A” for the course with very reasonable scores.

### Homework (HW)

<i>Homework</i>	<i>Possible</i>	<i>Earned</i>
Introduction	100	100
Section 1.1	100	100
Section 1.3	100	100
Section 1.4	100	95
Section 1.5	100	100
Section 1.6	100	100
Section 2.1A	100	80
Section 2.1B	100	100
Section 2.2	100	99
Section 2.3	100	100
Section 2.4	100	100
Section 2.5	100	95
Section 3.1	100	93
Section 3.2	100	100
Section 3.3	100	95
Section 3.5	100	93
<b>Total</b>	<b>1600</b>	<b>1550</b>

Divide the HW total by 160 (this example has 1550/160) and write the result here: 9.7 Put this result in the table below:

### Grading Scale

- A: 90 – 100 % (pts)
- B: 80 – 89% (pts)
- C: 70 – 79% (pts)
- D: 60 – 69% (pts)
- F: Below 50% (pts)

<b>TOTALS</b>	<i>Possible</i>	<i>Earned</i>
HW (average of all 16)	10	9.7
Test 1 (high score)	30	28.5
Test 2 (high score)	30	27.0
Test 3 (high score)	30	25.5
Sum of the scores above	100	90.7

## Lesson 1A

### Introduction

#### Lesson Objectives

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- Review selected algebra topics
- Determine your goals for the course

#### Overview

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Before beginning the first written assignment, refer to the *Course Rules* in Canvas for your instructor's assignment requirements. Assignments 1A is submitted through Canvas.

This lesson will cover selected review material and give me an idea about your goals for the course. The idea is to refresh your memory on a few topics and get you thinking about math again in case it has been a while since you completed a math course. Video lectures have been loaded into a "media assignment" in **MyLab**. The media assignment is supplementary and is not part of your grade, but may be helpful in completing this work. Work through each of the following tasks, carefully filling out the following pages.

#### Directions for Written Assignment (written assignment on pages 10–14)

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#### Section 1A

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##### 1. Solve an equation with fractions.

Do you remember how to solve a linear equation that has fractions? Try question 1 on the introductory assignment. If you need a little guidance, watch the first video in the mylabsplus introductory media assignment and take notes here.

##### **WORK INTRODUCTORY ASSIGNMENT EXERCISE #1**

See page 10 below.

##### 2. Solve a formula for a variable

Do you remember how to isolate a variable in a formula? Try question 2 on the introductory assignment. If you need a little guidance, watch the second video in the mylabsplus introductory media assignment and take notes here.

##### **WORK INTRODUCTORY ASSIGNMENT EXERCISE #2**

##### 3. Solve an equation by factoring

Do you remember how to solve an equation by factoring? Try question 3 on the introductory assignment. If you need a little guidance, watch the third video in the mylabsplus introductory media assignment and take notes here.

##### **WORK INTRODUCTORY ASSIGNMENT EXERCISE #3**

**4. Use the Pythagorean theorem to find the missing side of a right triangle**

Do you remember how use the Pythagorean theorem? Try question 4 on the introductory assignment. If you need a little guidance, watch the fourth video in the mylabsplus introductory media assignment and take notes here.

**WORK INTRODUCTORY ASSIGNMENT EXERCISE #4**

**5. Find the distance between two points**

Do you remember how to find the distance between two points? Try question 5 on the introductory assignment. If you need a little guidance, watch the fifth video in the mylabsplus introductory media assignment and take notes here.

**WORK INTRODUCTORY ASSIGNMENT EXERCISE #5**

**6. Consider how the distance formula and the Pythagorean theorem are related.**

If it seems a little unclear, then consider watching the fifth video in the mylabsplus media assignment. Explain your understanding of the relationship in question 6 of the introductory assignment.

**WORK INTRODUCTORY ASSIGNMENT EXERCISE #6**

**7. Reduce fractions**

Do you remember how to reduce a fraction by hand? Calculators are not allowed on the tests in this class, so it helps to review this process. If you don't quite remember all the details, then watch the sixth video in the mylabsplus media assignment and take notes here.

**WORK INTRODUCTORY ASSIGNMENT EXERCISE #7**

**8. Simplify radicals**

Do you remember how simplify a radical by hand? Calculators are not allowed on the tests in this class, so it helps to review this process. If you don't quite remember all the details, then watch the seventh video in the mylabsplus media assignment and take notes here.

**WORK INTRODUCTORY ASSIGNMENT EXERCISE #8**

**9. Writing and goals**

**WORK INTRODUCTORY ASSIGNMENT EXERCISE #9, 10, and 11**

Math 144 Introductory assignment

Name \_\_\_\_\_

This assignment is intended to refresh your memory about algebra and problem-solving skills that you will find helpful in trigonometry. There is an optional, non-graded “media assignment” in MyLabs with videos that will help you with this material. It will also help me read your handwriting in later work.

Show all steps for full credit.

1. Here is a linear equation with fractions. Remember, you can use the least common denominator to clear the fractions. Solve for the variable  $x$ :  $\frac{5}{4}x - \frac{7}{2} = \frac{31}{4}$

Answer:

2. Solve the formula for the variable  $C$ :  $AC + PT = DB$

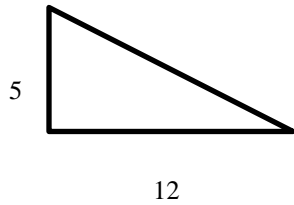
Answer:

3. Solve for the variable  $\theta$  (“theta”). Remember to look for the greatest common factor first and note that the last term is twelve times theta.

$$\theta^3 - 4\theta^2 - 12\theta = 0$$

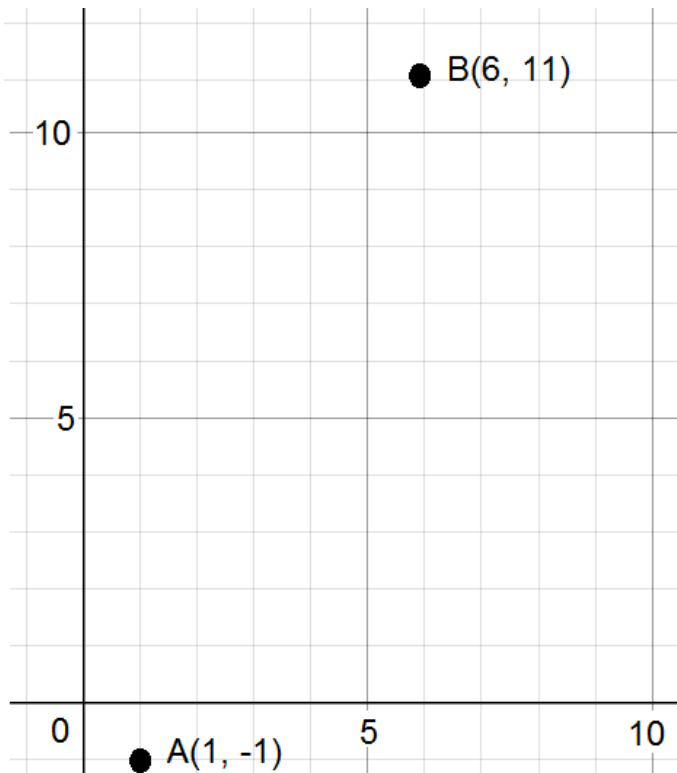
Answer:

4. Use the Pythagorean theorem to find the length of the hypotenuse of the following right triangle.



Answer:

5. Find the distance between the points shown in the following graph. You may look up the distance formula or use the information from question 4.



Distance:

6. How are exercises 4 and 5 related? Use complete sentences to answer.

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7. In this course, you frequently have to reduce fractions. Reduce the following fractions. (No work is needed, but I recommend that you complete these “by hand” without a calculator.) Use improper fractions instead of mixed numbers when they come up.

$$\frac{3}{6} =$$

$$\frac{6}{8} =$$

$$\frac{36}{30} =$$

$$\frac{14}{16} =$$

$$\frac{45}{50} =$$

8. In this course, you also have to simplify square roots. Give these a try. You will find it helpful to write some intermediate products under the radicals.

$$\sqrt{28} =$$

$$\sqrt{48} =$$

9. It can be hard for me to read people’s handwriting. Please bear with me and copy the following “classic” sentence. “The quick red fox jumped over the lazy brown dog.”

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Now print the sentence all in capital letters:

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10. There are a few Greek letters that are commonly used in trigonometry. You may or may not have seen them before, and it will help me read your handwriting if you copy them for me.

Name	Character	Your copy:
“alpha”	$\alpha$	_____
“beta”	$\beta$	_____
“gamma”	$\gamma$	_____
“pi” (lower case)	$\pi$	_____
“pi” (upper case)	$\Pi$	_____
“theta”	$\theta$	_____
“phi”	$\varphi$	_____

11. Finally, tell me why you are taking this course and share something about your academic and career goals.

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## Lesson 1B

### Chapter 1: Section 1.1

#### Lesson Objectives

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- Understanding Degree Measure
- Finding Coterminal Angles Using Degree Measure
- Understanding Radian Measure
- Converting between Degree Measure and Radian Measure
- Finding Coterminal Angles Using Radian Measure

#### Overview

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This lesson will cover Section 1.1 in your e-Text. Work through each of the following tasks, carefully filling out the following pages.

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#### Section 1.1 An Introduction to Angles: Degree and Radian Measure

- Work through TTK #1 then do problem #1
- Work through Objective 1 then do problems #2-3
- Work through Objective 2 then do problems #4-5
- Work through Objective 3 then do problems #6-9
- Work through Objective 4 then do problems #10-14
- Work through Objective 5 then do problems #15-16

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#### Section 1.1 an Introduction to Angles: Degree and Radian Measure

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##### 1.1 Things To Know

##### 1. Sketching the Graph of a Circle

Can you sketch the graph of a circle? Try working through a “You Try It” problem or watch the video.

## WORK SECTION 1.1 HW EXERCISE #1

### Section 1.1 Introduction

What is the definition of a **vertex**?

What is the definition of the **initial side**?

What is the definition of the **terminal side**?

Sketch an angle with positive measure, labeling the vertex, initial side, and terminal side. Do the same for an angle with negative measure.

What does it mean for an angle to be in **standard position**?

Sketch an angle in standard position having positive measure.

Sketch an angle in standard position having negative measure.

## Section 1.1 Objective 1 Understanding Degree Measure \_\_\_\_\_

In the **degree measure** system, what is the symbol used to indicate a degree? How many degrees are in a one complete counterclockwise rotation?

Sketch three coordinate planes, illustrating angles of 360, 90, and -45 degrees respectively. (See Figures 3, 4, and 5.)

What is the definition of an **acute angle**?

What is the definition of an **obtuse angle**?

What is the definition of a **quadrantal angle**?

What is the term for an angle of exactly 90 degrees?

What is the term for an angle of exactly 180 degrees?

What does it mean for angles to be **coterminal**?

Sketch the two coordinate planes illustrating common positive and negative angles as seen in Figure 6.

Work through the video accompanying Example 1 showing all work below.

Draw each angle in standard position and state the quadrant in which the terminal side of the angle lies or the axis on which the terminal side of the angle lies.

a.  $\theta = 60^\circ$

b.  $\alpha = -270^\circ$

c.  $\beta = 420^\circ$

## WORK SECTION 1.1 HW EXERCISES #2-3

### Section 1.1 Objective 2 Finding Coterminal Angles Using Degree Measure

What is the definition of **Coterminal Angles**?

Starting with a given angle, how can you obtain coterminal angles? (See the **coterminal angle** definition box.)

What notation is used to denote the angle of least nonnegative measure that is coterminal with  $\theta$ ?

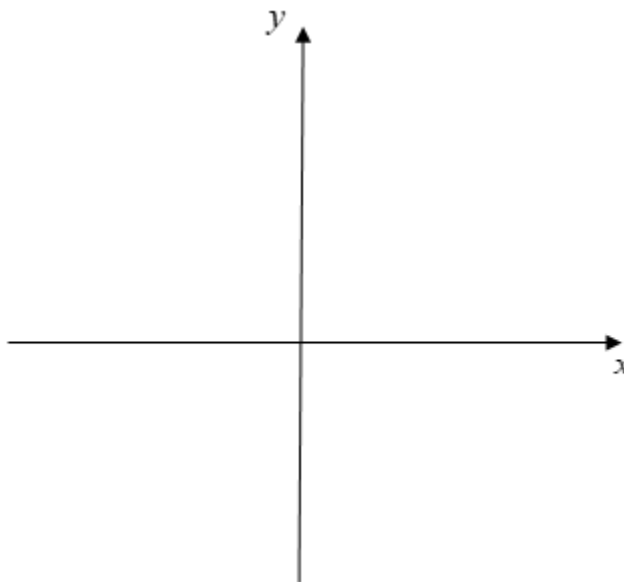
Work through the video with Example 2 and show all work below.

Find the angle of least nonnegative measure,  $\theta_C$ , that is coterminal with  $\theta = -697^\circ$ .

**NOW WORK SECTION 1.1 HW EXERCISES #4-5****Section 1.1 Objective 3 Understanding Radian Measure**

Carefully work through the **animation** seen next to Objective 3 on page 1.1-13 and answer the questions below:

Draw a circle centered at the origin having a radius of  $r$  units. What is the equation of the circle?



What is the definition of a **central angle**?

What is the definition of an **intercepted arc**? What variable is typically used to represent it?

On the graph of your circle, above, draw a central angle so that the intercepted arc is the same length as the radius of your circle.

What is the measure of this central angle called?

What is the definition of a **radian**?

Approximately how many radians are there in a circle?

Carefully work through the **animation** seen near the bottom of page 1.1-13 and answer the questions below:

What is the formula for the circumference of a circle of radius  $r$  units?

(Fill in the blank) A central angle of  $360^\circ$  intercepts an arc length of \_\_\_\_\_.

Complete the proportion below as seen in the animation.

$$\frac{360^\circ}{\square} = \frac{\square}{r}$$

Fill in the box:  $360^\circ = \square$  radians.

Fill in the box:  $180^\circ = \square$  radians.

Sketch three coordinate planes, illustrating angles of  $2\pi$ ,  $\frac{\pi}{2}$ , and  $\frac{-\pi}{4}$  radians respectively. (See Figures 10, 11, and 12.)

Sketch two coordinate planes illustrating common positive and negative angles in radians as seen in Figure 13.

Work through the interactive video accompanying Example 3 showing all work below.

Draw each angle in standard position and state the quadrant in which the terminal side of the angle lies or the axis on which the terminal side of the angle lies.

a.  $\theta = \frac{\pi}{3}$

b.  $\alpha = -\frac{3\pi}{2}$

c.  $\beta = \frac{7\pi}{3}$

WORK SECTION 1.1 HW EXERCISES #6-9 AND SUBMIT FOR GRADING.

### WORK SECTION 1.1 HW EXERCISES #6-9

#### Section 1.1 Objective 4 Converting between Degree Measure and Radian Measure

To convert **degrees to radians**, multiply by \_\_\_\_\_.

To convert **radians to degrees**, multiply by \_\_\_\_\_.

Work through the interactive video with Example 4 and show all work below.

Convert each angle given in degree measure into radians.

a.  $45^\circ$

b.  $-150^\circ$

c.  $56^\circ$

**WORK SECTION 1.1 HW EXERCISES #10-12**

Work through the interactive video with Example 5 and show all work below.

Convert each angle given in radian measure into degrees. Round to two decimal places if needed.

a.  $\frac{2\pi}{3}$  radians

b.  $-\frac{11\pi}{6}$  radians

c. 3 radians

**WORK SECTION 1.1 HW EXERCISES #13-14****Section 1.1 Objective 5 Finding Coterminal Angles Using Radian Measure**

For any angle  $\theta$  and for any nonzero integer  $k$ , we can find a coterminal angle using what expression?

Work through Example 6 and show all work below.

Find three angles that are coterminal with  $\theta = \frac{\pi}{3}$  using  $k = 1$ ,  $k = -1$ , and  $k = -2$ .



Work through the video with Example 7 and show all work below.

Find the angle of least nonnegative measure,  $\theta_c$ , that is coterminal with  $\theta = -\frac{21\pi}{4}$ .

**WORK SECTION 1.1 HW EXERCISES #15-16**