



**The equation of state of neutron-rich matter  
at fourth order of chiral effective field theory  
and  
the radius of a medium-mass neutron star**

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**IWNT39-2022**

**Supported by the U.S. Department of Energy**



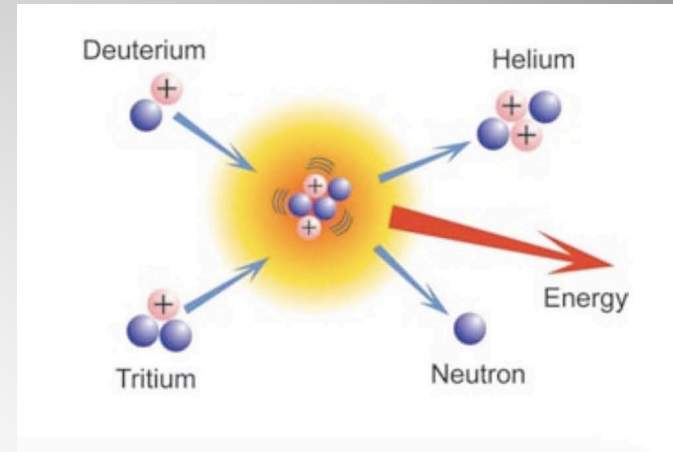
# Outline



- ❑ A few basic facts about neutron stars (NS)
- ❑ Nuclear physics of NS:
  - Overview of the most important nuclear physics ingredients:
    - ❖ Neutron matter (NM), symmetric nuclear matter (SNM), the symmetry energy and its density dependence
    - ❖ The role of the Equation of state (EoS)
- ❑ Predictions & constraints
  - Ab initio vs. phenomenology
  - NS radii and the thickness of the neutron skin
- ❑ Conclusions and outlook

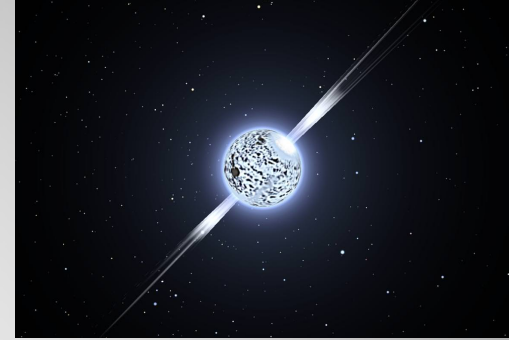
## SOME FACTS IN A NUTSHELL:

- ❑ When a star fuses its way to  $^{56}\text{Fe}$ , the fuel is gone
- ❑ The iron core accumulates to about 1.4 solar masses
- ❑ The electron degeneracy pressure that had been supporting the star against gravity gives up
- ❑ The star collapses inward
- ❑ Under the pressure involved in the collapse, protons and electrons combine to form neutrons plus neutrinos
- ❑ The neutrinos escape, and the neutrons settle to become a neutron star, with neutron degeneracy and neutron-neutron forces opposing gravity.

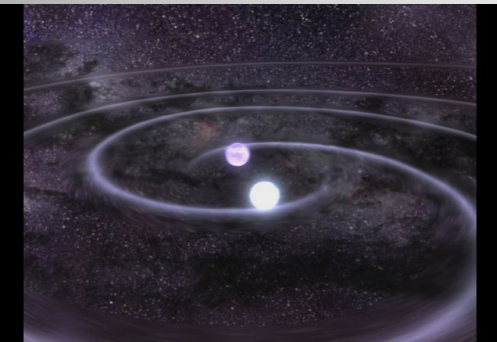


*“With all reserve we advance the view that supernovae represent the transition from ordinary stars into neutron stars, which in their final stages consist of closely packed neutrons.”* **W. Baade and F. Zwicky, 1933.**

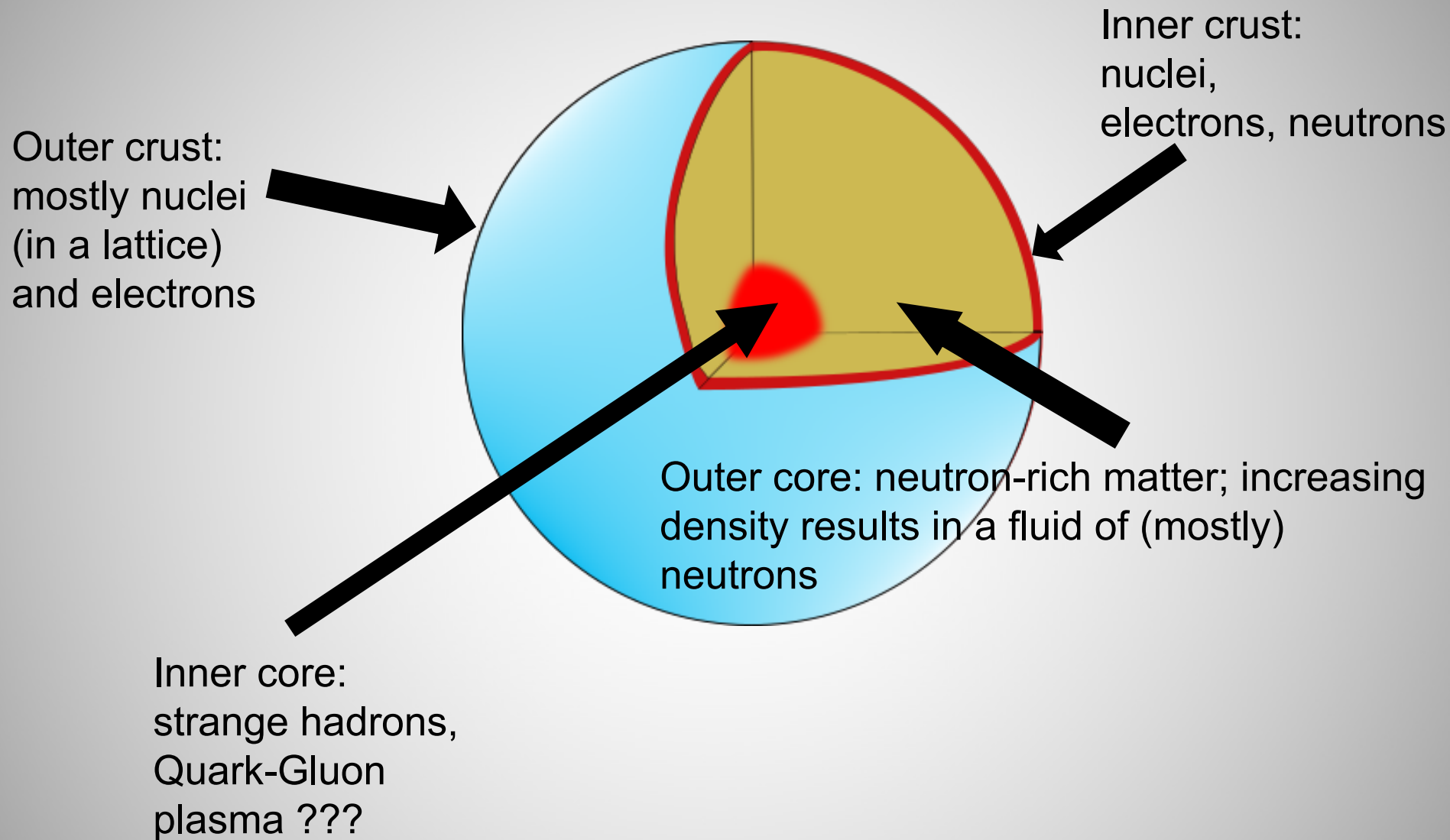
## NS are unique systems:



- ❑ A wonderful mix of all fundamental forces in nature is represented in a neutron star.
  
- ❑ Opportunity to study matter under conditions beyond lab possibilities...  
A tablespoon of neutron star matter would weigh more than **1 billion tons** (the weight of Mount Everest).



Let's now focus on a mature star, which has recovered from its birth trauma.



## TOV equations for spherical symmetry

$$\frac{dP}{dr} = - \frac{G\epsilon(r)M(r)}{c^2 r^2} \left(1 + \frac{P(r)}{\epsilon(r)}\right) \left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right) \left(1 - \frac{2GM(r)}{rc^2}\right)^{-1}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \epsilon(r)$$

$\epsilon$  = energy density

$P$  = pressure

$\epsilon(p)$  is obtained from the equation of state (EoS) for stellar matter

# Neutron-rich matter

Energy per particle in  
isospin asymmetric  
matter:

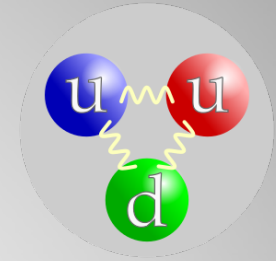
$$e(\rho, \alpha) = e(\rho, 0) + e_{sym} \alpha^2$$

$$e_{sym}(\rho) = e(\rho, 1) - e(\rho, 0) \quad \alpha = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Expansion of the symmetry energy about saturation density:

$$e_{sym}(\rho) = e_{sym}(\rho_0) + L \frac{\rho - \rho_0}{3\rho_0} + \frac{K}{2} \frac{(\rho - \rho_0)^2}{(3\rho_0)^2} \dots$$

Our (still incomplete) knowledge of the nuclear force is the result of decades of struggle.



**Nuclear Forces: A hierarchy of scales.**

**Scale determines the appropriate degrees of freedom.**

**This concept is central to the development of an Effective Field Theory.**

**Presently:**

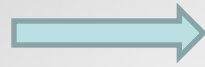
**For low energies, nuclear chiral Effective Field Theory (EFT) has become the most favorable approach to construct nuclear two- and few-body forces in a systematic way.**

**Based upon the symmetries of low-energy QCD, while using degrees of freedom relevant for low-energy nuclear physics. Predictions can be improved systematically.**

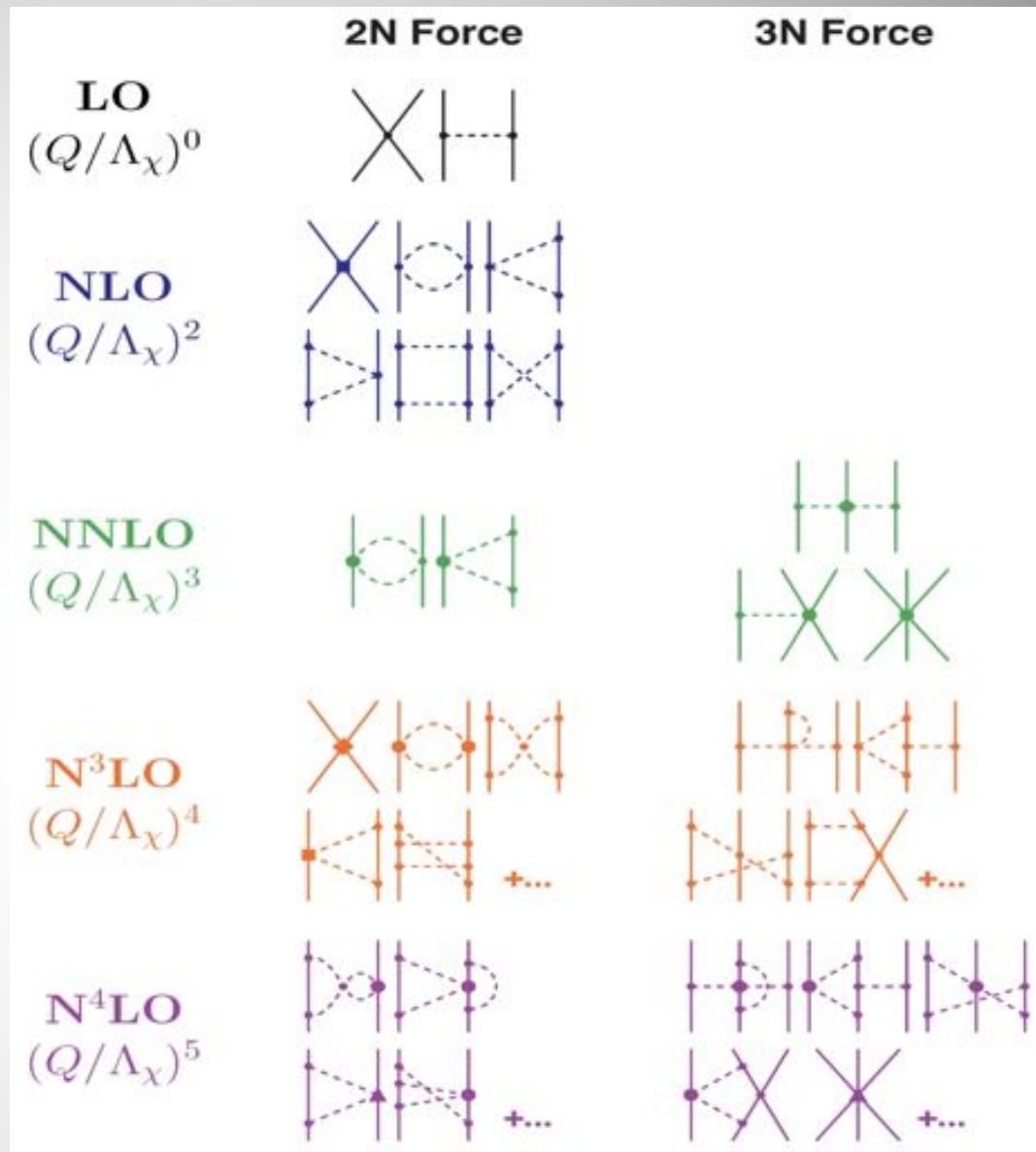
**Together with an organizational scheme to rank-order the various diagrams (power counting),  
nuclear two- and few-body forces can be developed  
in a controlled hierarchy.**



# Hierarchy of nuclear forces in Chi-EFT

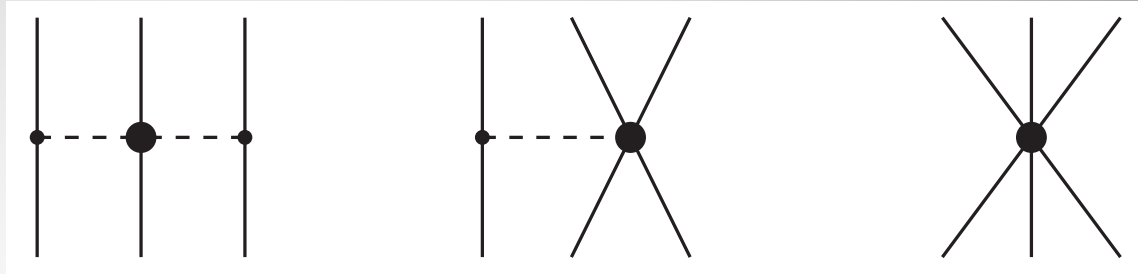


We include all two- and three-nucleon forces up to fourth order.

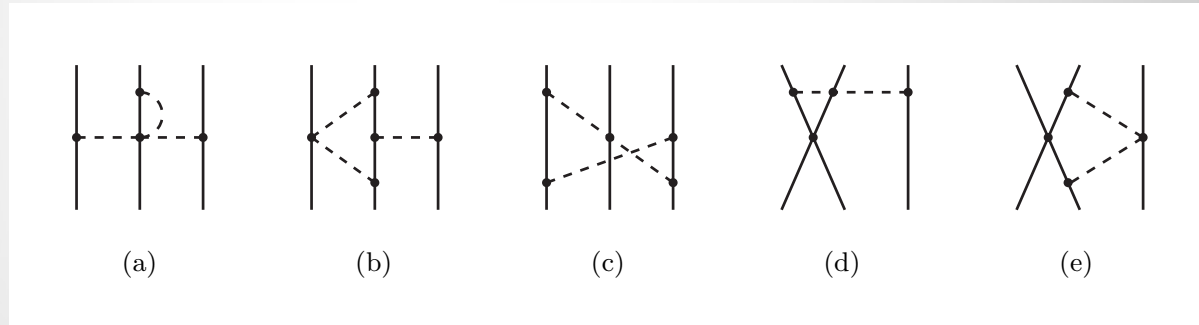


**2NF: high-quality NN chiral potentials at  $N^3LO$**   
(Entem, Machleidt, and Nosyk, 2017)

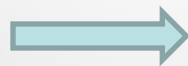
**3NF**



**Subleading 3NF: Some typical diagrams:**



These free-space interactions



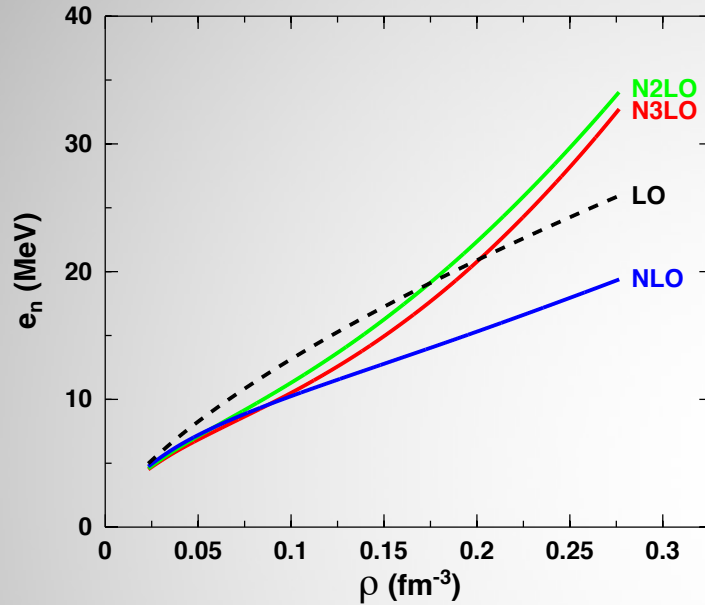
nonperturbative particle-particle ladder  
self-consistent single-particle spectrum

F.S. & Randy Millerson, PRC104, 034308

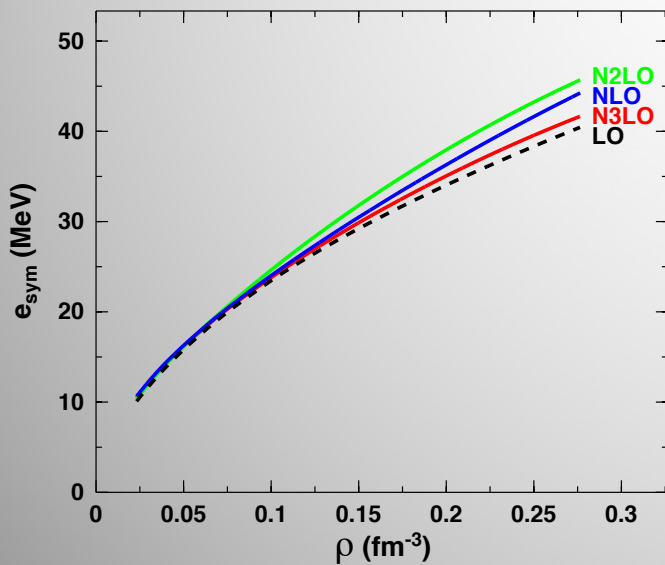
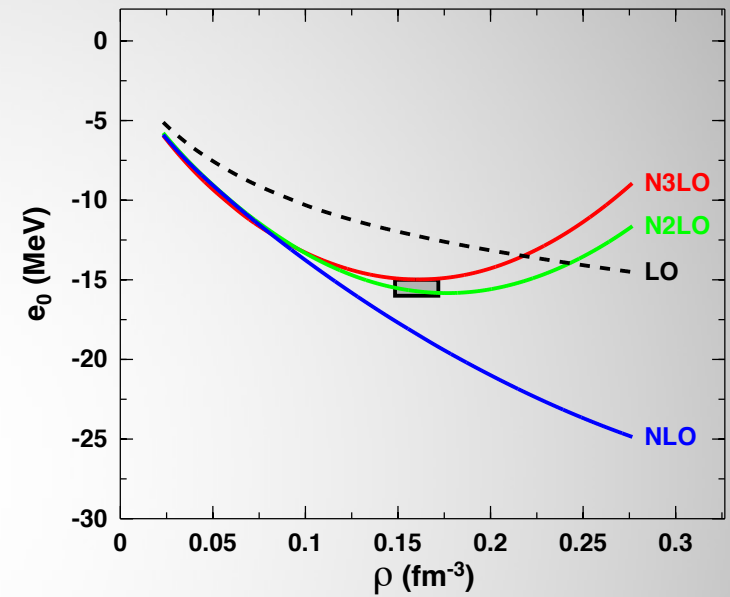
F.S. & Randy Millerson, PRC104, 064312

# Ab initio predictions (F.S. et al.)...

... for the EoS of NM,  $e_n$



...and for the EoS of SNM,  $e_0$



...and for the symmetry energy,  $e_{\text{sym}}$

Recall:

$$e(\rho, \alpha) = e_0(\rho) + e_{\text{sym}}(\rho) \alpha^2 \quad 11$$

**Chiral Effective Field Theory is a low-energy/momentum theory. Thus our chiral EoS is applicable within a limited range of Fermi momenta.**

**The complete EoS is created by matching three contributions:**

**Subnuclear  
crystal EoS**

**U**

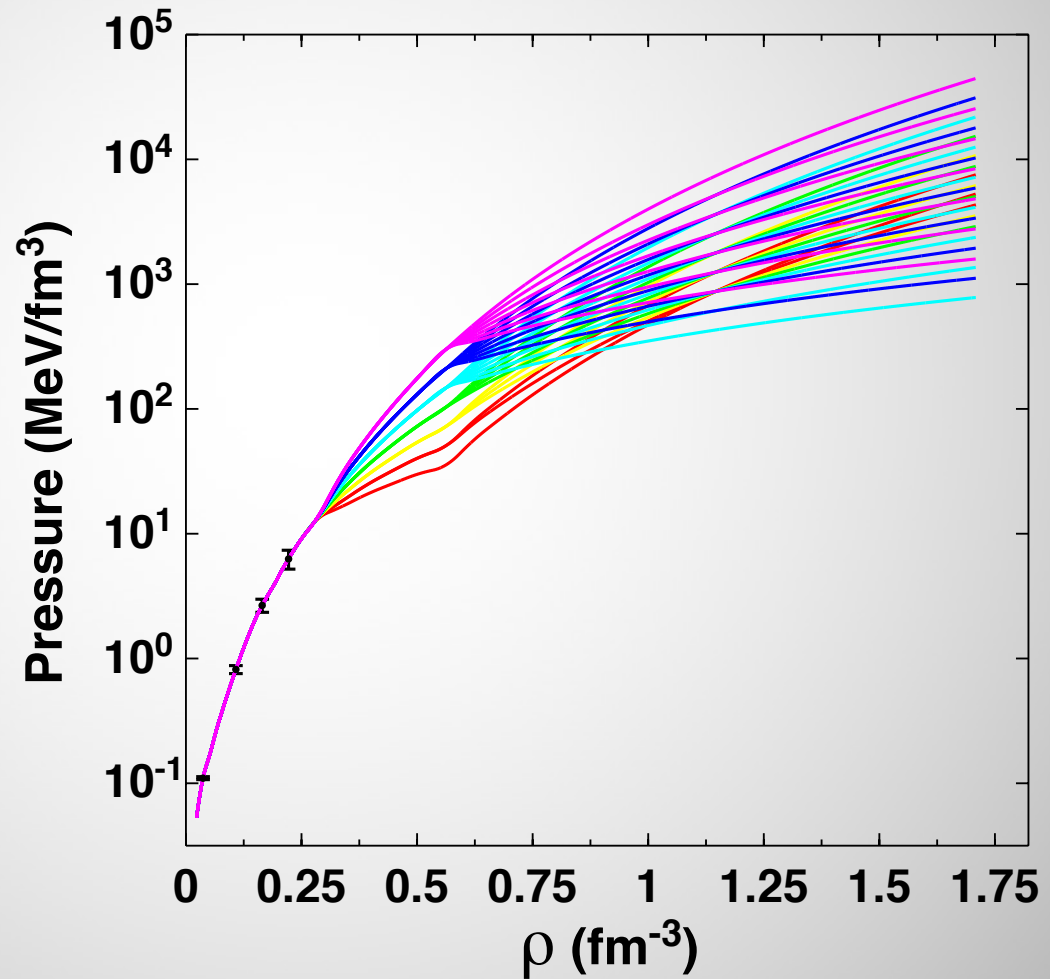
**Microscopic  
EoS in beta  
equilibrium**

**U**

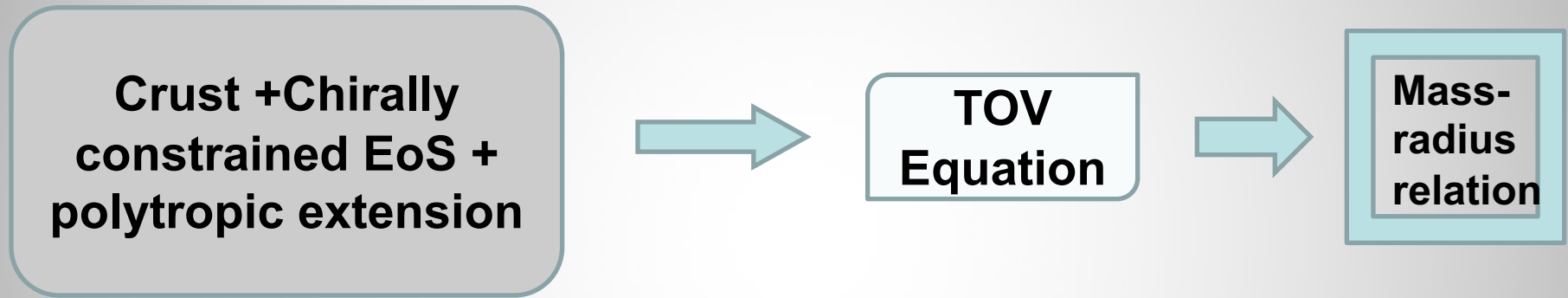
**High density  
continuation  
(?)**

## High-density continuation with piecewise polytropes

$$P(\rho) = \alpha \rho^\gamma$$



# Neutron star Mass and Radius



**Constraints:** Maximum mass at least 2.01 solar masses\*

**Causality constraint,  $v_s < c$**

\*Miller, M.C. et al. The Radius of PSR J0740+6620 from NICER and XMM-Newton Data.  
arXiv:2105.06979 [astro-ph.HE]

## Predictions (F.S. *et al.*)

$$e_{\text{sym}}(\rho_0) = 31.3 \pm 0.8 \text{ MeV}$$

$$L = 52.6 \pm 4.0 \text{ MeV}$$

correlated



$$R_{1.4} = 11.96 \pm 0.80 \text{ km}$$

**L is strongly correlated with another observable, measurable in terrestrial experiments.**

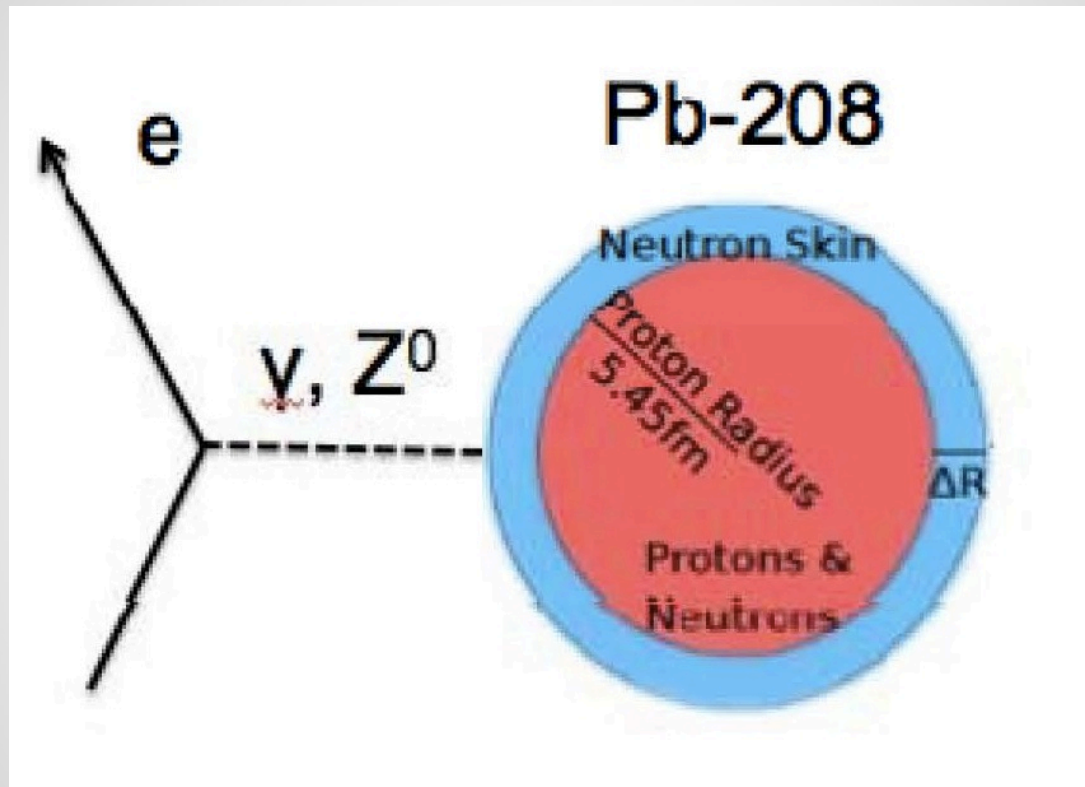
## The radius of the average-mass NS is only weakly sensitive to the high-density continuation

A characteristic example:

1 <sup>st</sup> polytropic index	2 <sup>nd</sup> polytropic index	R(km)	central density (fm <sup>-3</sup> )	causal?
2.5	3.5	11.34	0.63	Y
	4.0	11.34	0.62	Y
	4.5	11.35	0.61	Y
3.0	3.0	11.87	0.51	Y
	3.5	11.87	0.51	Y
	4.0	11.87	0.51	Y
	4.5	11.87	0.51	Y



**PREX II (Parity Radius Experiment, Jlab, 2021)**  
**(Reed, Fattoyev, Horowitz, Piekarewicz)**  
**(exploits parity-violating electron scattering to measure the neutron skin of  $^{208}\text{Pb}$ )**



Our predictions:  
(PRC105, 064303 (2022))

$$\mathbf{S = 0.13 - 0.17 \text{ (fm)}}$$

*...This result challenges myriad of experimental measurements and theoretical predictions....*

arXiv:2101.03193, (Reed, Fattoyev, Horowitz, & Piekarewicz)

**PREX II values:**

*...This result challenges our present understanding of the density dependence of the symmetry energy extracted from various experimental and theoretical analyses....  
(PRL126, 172503 (2021))*

*Or rather: myriad of experimental measurements and microscopic theoretical predictions do not support this result...*

$$J = 38.09 \pm 4.73 \text{ MeV}$$

$$L = 106 \pm 37 \text{ MeV}$$

$$S = 0.29 \pm 0.07 \text{ fm}$$

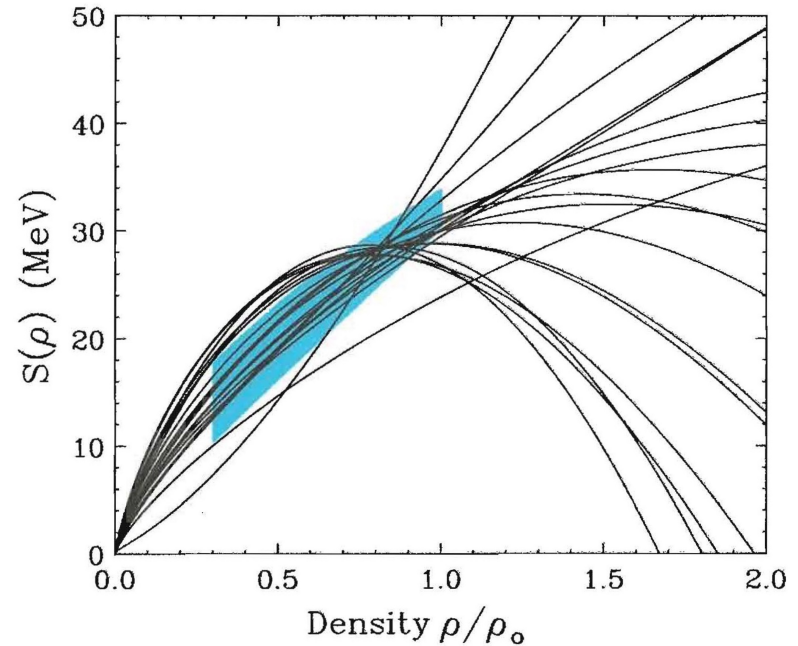
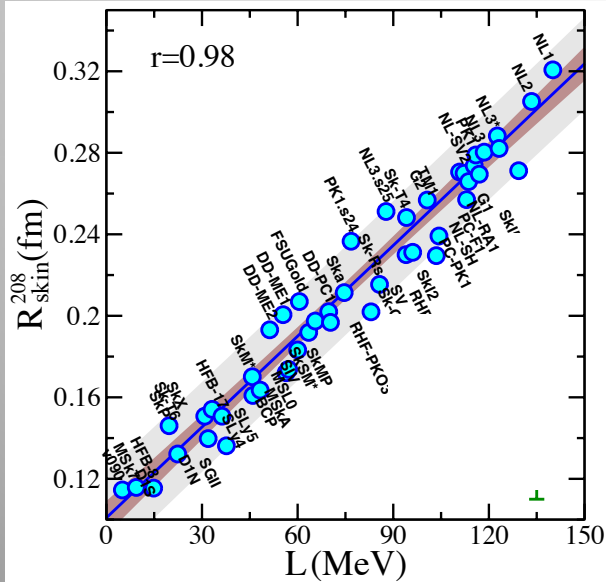
$$R_{1.4} = (13.33 - 14.26) \text{ km}$$

$$J = e_{\text{sym}}(\rho_0)$$

Are there predictions that agree with PREX II results?

YES, from phenomenological models...

Phenomenology



Symmetry energy predicted by some of the many parameterizations of the Skyrme model. 19

From a Bayesian analysis of **300,000** EoS constrained by microscopic nuclear theory and nuclear experiments [Lin & Holt, EPJ 55, 209 (2019)]

$R_{1.4} = (11.36 - 12.48) \text{ km}$        $1\sigma$  confidence level

$R_{1.4} = (10.26 - 12.87) \text{ km}$        $2\sigma$  confidence level

## Conclusions and outlook:

- Even though chiral EFT cannot reach out to the extreme-density and yet unknown regimes at the core of these remarkable systems, continuously improved *ab initio* calculations of the nuclear EoS are an essential foundation for interpreting current and future observations in terms of microscopic nuclear forces.
  - ❑ Our predictions are characteristic of EFT results based on high quality 2NF and realistic (leading and subleading) 3NF.
  - ❑ Recent laboratory constraints based on weak electron scattering cannot be reconciled with any of the *ab initio* theoretical predictions.
  - ❑ See Hu *et al.* [arXiv:2112.01125](https://arxiv.org/abs/2112.01125) for how low-energy NN data constraint the value of  $L$ .
- A fully microscopic EoS up to central densities of the most massive stars—potentially involving non-nucleonic degrees of freedom—is not within reach.
- Nevertheless, neutron stars are powerful natural laboratories for constraining theories of EoS. One must be mindful about the theory's limitations and the best ways to extract useful information from the observational constraints.

*Thank you*

*...and see you next year!*